Cuadratura Gausiana

parametros:

n=nodos
grado $\leq (2n-1)$
$\int_{-1}^{1} f(x)dx = \sum_{i=1}^{n}(w_i * f(x_i))$

n=2 ; grados≤ 3
$f(x) = 1- >\int_{-1}^{1} dx = w_1 f(x_1) + w_2 f(x_2) = w_1 + w_2 = x|_{-1}^{1} = 1 - (-1) = 2$
$f(x) = x- > \int_{-1}^{1} x.dx = w_1 f(x_1) + w_2 f(x_2) = w_1 x_1 + w_2 x_2 = \frac{x^2}{2}|_{-1}^{1} = 0$
$f(x) = x^2- > \int_{-1}^{1} x^2.dx = w_1 f(x_1) + w_2 f(x_2) = w_1 x_1^2 + w_2 x_2^2 = \frac{x^3}{3}|_{-1}^{1} = \frac{2}{3}$
$f(x) = x^3- > \int_{-1}^{1} x^3.dx = w_1 f(x_1) + w_2 f(x_2) = w_1 x_1^3 + w_2 x_2^3 = \frac{x^4}{4}|_{-1}^{1} = 0$

n=3 ; grados≤ 5
$f(x) = 1- >\int_{-1}^{1} dx = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) = w_1 + w_2 + w_3 = x|_{-1}^{1} = 1 - (-1) = 2$
$f(x) = x- > \int_{-1}^{1} x.dx = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) = w_1 x_1 + w_2 x_2 + w_3 x_3 = \frac{x^2}{2}|_{-1}^{1} = 0$
$f(x) = x^2- > \int_{-1}^{1} x^2.dx = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) = w_1 x_1^2 + w_2 x_2^2 + w_3 x_3^2 = \frac{x^3}{3}|_{-1}^{1} = \frac{2}{3}$
$f(x) = x^3- > \int_{-1}^{1} x^3.dx = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) = w_1 x_1^3 + w_2 x_2^3 + w_3 x_3^3 = \frac{x^4}{4}|_{-1}^{1} = 0$
$f(x) = x^4- > \int_{-1}^{1} x^3.dx = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) = w_1 x_1^4 + w_2 x_2^4 + w_3 x_3^4 = \frac{x^5}{5}|_{-1}^{1} = \frac{2}{5}$
$f(x) = x^5- > \int_{-1}^{1} x^3.dx = w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) = w_1 x_1^5 + w_2 x_2^5 + w_3 x_3^5 = \frac{x^6}{6}|_{-1}^{1} = 0$

Ejemplos
$\int_{-1}^{1} f(x)dx = \sum_{i=1}^{n}(w_i * f(x_i))$

n=2 ->
$\int_{-1}^{1} f(x)dx = \sum_{i=1}^{n}(w_i * f(x_i)) = w_1 * f(x_1) + w_2 * f(x_2) = 1 * f(0,557) + 1 * f(-0,557)$

$\int_{-1}^{1}(3x^4 + 3x^2 + 7) = 17,2- > 1 * f(0,557) + 1 * f(-0,557)$
$\int_{-1}^{1}(3x^4 + 3x^2 + 7) = 3\frac{x^5}{5} + 3\frac{x^3}{3} + 7x|_{-1}^{1} = 17,2$

n=3->
$\int_{-1}^{1} f(x)dx = \sum_{i=1}^{n}(w_i * f(x_i)) = w_1 * f(x_1) + w_2 * f(x_2) + w_3 * f(x_3) = 0,556 * f(0,7746) + 0,556 * f(-0,7746) + 0,889 * f(0)$

Teorema de translación

$t = \frac{b-a}{2}x + \frac{a+b}{2}$; $dt = \frac{b-a}{2}dx$
$\int_a^b f(t)dt \approx \int_{-1}^1 f(\frac{b-a}{2}x + \frac{a+b}{2})(\frac{b-a}{2})dx = \frac{b-a}{2}\sum_{i=1}^n w_i f(\frac{b-a}{2}x_i + \frac{a+b}{2})$
$\int_{-1}^1 f(\frac{b-a}{2}x + \frac{a+b}{2})(\frac{b-a}{2})dx$
$\int_{-1}^1 f(x)dx \approx \sum_{i=1}^2 (w_i * f(x_i)) = w_1 f(\frac{b-a}{2}x_1 + \frac{a+b}{2})(\frac{b-a}{2}) + w_2 f(\frac{b-a}{2}x_2 + \frac{a+b}{2})(\frac{b-a}{2})$

Ejemplo: calcular la integral definida de $\int_1^{1,5} e^{-x^2}dx$ para n=2
$\frac{b-a}{2} = \frac{1,5-1}{2} = 0,25$
$\frac{b+a}{2} = \frac{1,5+1}{2} = 1,25$

$f(x) = e^{-x^2}$ para $n = 2$ ->
$\int_1^{1,5} e^{-x^2}dx = \frac{b-a}{2}\sum_{i=1}^2 w_i f(\frac{b-a}{2}x_i + \frac{a+b}{2}) = 0,25(w_1 * f(0,25x_1 + 1,25) + w_2 * f(0,25x_2 + 1,25)) =$
$0,25(f(0,25*0,5774+1,25)+f(0,25*-0,5774+1,25)) = 0,25(f(1,39435)+f(1,10565)) = 0,25(e^{-(1,39435)^2} + e^{-(1,10565)^2})$
$= 0,25(0,1430 + 0,1095) = 0,10940104496734$