Polinomio de Taylor

Idea: ayuda a predecir el valor de una funcion en un punto x, en terminos de la funcion y sus derivadas en otro punto.

La funcion debe ser suave o diferenciable en cualquier orden, es decir todas sus derivadas son continuas en sus ordenes

el polinomio de Taylor debe coincidir con $f(x)$ y con sus n primeras derivadas.

si el punto de considencia fuera $a = 0$, el polinomio estaria dado por:

$$p(x) = \sum_{k=0}^{n} (\frac{x^k}{k!}) f^k(0)$$

Si se translada el origien y $a \neq 0$, se tiene que:

$$p(x) = \sum_{k=0}^{n} \frac{(x-a)^k}{k!} f^k(a)$$

Ejemplos:

calcular el primer grado y tercer grado del polinomio de Taylor para $a = 0$:

$$f(x) = 5x^4 - 2x^3 + 3x^2$$

$$f'(x) = f^1(x) = 20x^3 - 6x^2 + 6x$$

$$f^2(x) = 60x^2 - 12x + 6$$

$$f^3(x) = 120x - 12$$

Para n= 1(grado 1) $p(x) = \sum_{k=0}^{n} \frac{(x-a)^k}{k!} f^k(a) - > p(x) = \sum_{k=0}^{1} \frac{x^k}{k!} f^k(0) = \frac{x^0}{0!} f(0) + \frac{x}{1!} f^1(0) = 0$

para n=3(grado 3) $p(x) = \sum_{k=0}^{n} \frac{(x-a)^k}{k!} f^k(a) - > p(x) = \sum_{k=0}^{3} \frac{x^k}{k!} f^k(0) = \frac{x^0}{0!} f(0) + \frac{x}{1!} f^1(0) + \frac{x^2}{2!} f^2(0) + \frac{x^3}{3!} f^3(0)$

$\frac{x^2}{2}(6) + \frac{x^3}{6}(-12) = 3x^2 - 2x^3$

calcular el primer grado y tercer grado del polinomio de Taylor para $a = \Pi/2$

$$f(x) = sen(x) - > f(\frac{\Pi}{2}) = sen(\frac{\Pi}{2}) = 1$$

$$f^1(x) = cos(x) - > f^1(\frac{\Pi}{2}) = cos(\frac{\Pi}{2}) = 0$$

$$f^2(x) = -sen(x)->f^2(\frac{\Pi}{2}) = -sen(\frac{\Pi}{2}) = -1$$

$$f^3(x) = -cos(x)->f^3(\frac{\Pi}{2}) = -cos(\frac{\Pi}{2}) = 0$$

n= 1 $p(x) = \sum_{k=0}^{n} \frac{(x-a)^k}{k!} f^k(a)->\sum_{k=0}^{1} \frac{(x-\frac{\Pi}{2})^k}{k!} f^k(\frac{\Pi}{2}) = \frac{(x-\frac{\Pi}{2})^0}{0!} f(\frac{\Pi}{2}) + \frac{(x-\frac{\Pi}{2})^1}{1!} f^1(\frac{\Pi}{2}) = 1 + \frac{(x-\frac{\Pi}{2})}{1!}(0) = 1$

n= 3 $p(x) = \sum_{k=0}^{n} \frac{(x-a)^k}{k!} f^k(a)->\sum_{k=0}^{3} \frac{(x-\frac{\Pi}{2})^k}{k!} f^k(\frac{\Pi}{2}) = \frac{(x-\frac{\Pi}{2})^0}{0!} f(\frac{\Pi}{2}) + \frac{(x-\frac{\Pi}{2})^1}{1!} f^1(\frac{\Pi}{2}) + \frac{(x-\frac{\Pi}{2})^2}{2!} f^2(\frac{\Pi}{2}) + \frac{(x-\frac{\Pi}{2})^3}{3!} f^3(\frac{\Pi}{2}) =$

$1 + 0 + \frac{(x^2+2(x)(\frac{\Pi}{2})+(\frac{\Pi}{2})^2)}{2}(-1) + \frac{(x^3+3x^2(-\frac{\Pi}{2})+3x(-\frac{\Pi}{2})^2+(-\frac{\Pi}{2})^3)}{6}(0) = 1 - \frac{x^2}{2} - \frac{\Pi x}{2} - \frac{\Pi^2}{8}$

calcular el primer grado y tercer grado del polinomio de Taylor para $a = \Pi/2$

$$f(x) = cos(x)->f(\frac{\Pi}{2}) = cos(\frac{\Pi}{2}) = 0$$

$$f^1(x) = -sen(x)->f^1(\frac{\Pi}{2}) = -sen(\frac{\Pi}{2}) = -1$$

$$f^2(x) = -cos(x)->f^2(\frac{\Pi}{2}) = -cos(\frac{\Pi}{2}) = 0$$

$$f^3(x) = sen(x)->f^3(\frac{\Pi}{2}) = sen(\frac{\Pi}{2}) = 1$$

n= 1 $p(x) = \sum_{k=0}^{n} \frac{(x-a)^k}{k!} f^k(a)->\sum_{k=0}^{1} \frac{(x-\frac{\Pi}{2})^k}{k!} f^k(\frac{\Pi}{2}) = \frac{(x-\frac{\Pi}{2})^0}{0!} f(\frac{\Pi}{2}) + \frac{(x-\frac{\Pi}{2})^1}{1!} f^1(\frac{\Pi}{2}) = 0 + \frac{(x-\frac{\Pi}{2})}{1!}(-1) = -x + \frac{\Pi}{2}$

n= 3 $p(x) = \sum_{k=0}^{n} \frac{(x-a)^k}{k!} f^k(a)->\sum_{k=0}^{3} \frac{(x-\frac{\Pi}{2})^k}{k!} f^k(\frac{\Pi}{2}) = \frac{(x-\frac{\Pi}{2})^0}{0!} f(\frac{\Pi}{2}) + \frac{(x-\frac{\Pi}{2})^1}{1!} f^1(\frac{\Pi}{2}) + \frac{(x-\frac{\Pi}{2})^2}{2!} f^2(\frac{\Pi}{2}) + \frac{(x-\frac{\Pi}{2})^3}{3!} f^3(\frac{\Pi}{2}) =$

$-x + \frac{\Pi}{2} + \frac{(x^2+2(x)(\frac{\Pi}{2})+(\frac{\Pi}{2})^2)}{2}(0) + \frac{(x^3+3x^2(-\frac{\Pi}{2})+3x(-\frac{\Pi}{2})^2+(-\frac{\Pi}{2})^3)}{6}(1) = 0.167x^3 - 0.785x^2 + 0.234x + 0.925$

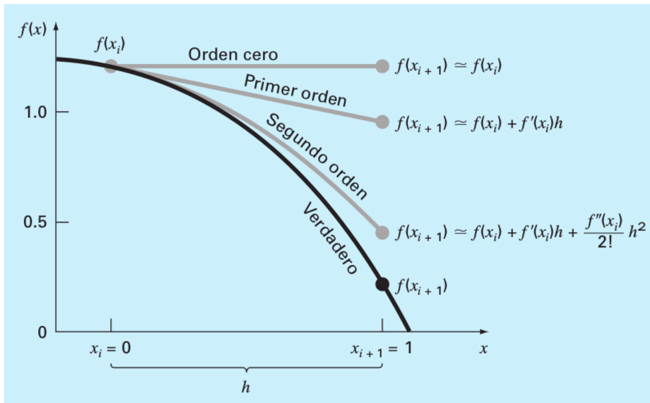

Figure 1: Aproximación por serie de Taylor polinomio 2 grados.Suponga que $a = x_1$;$x = x_{i+1}$

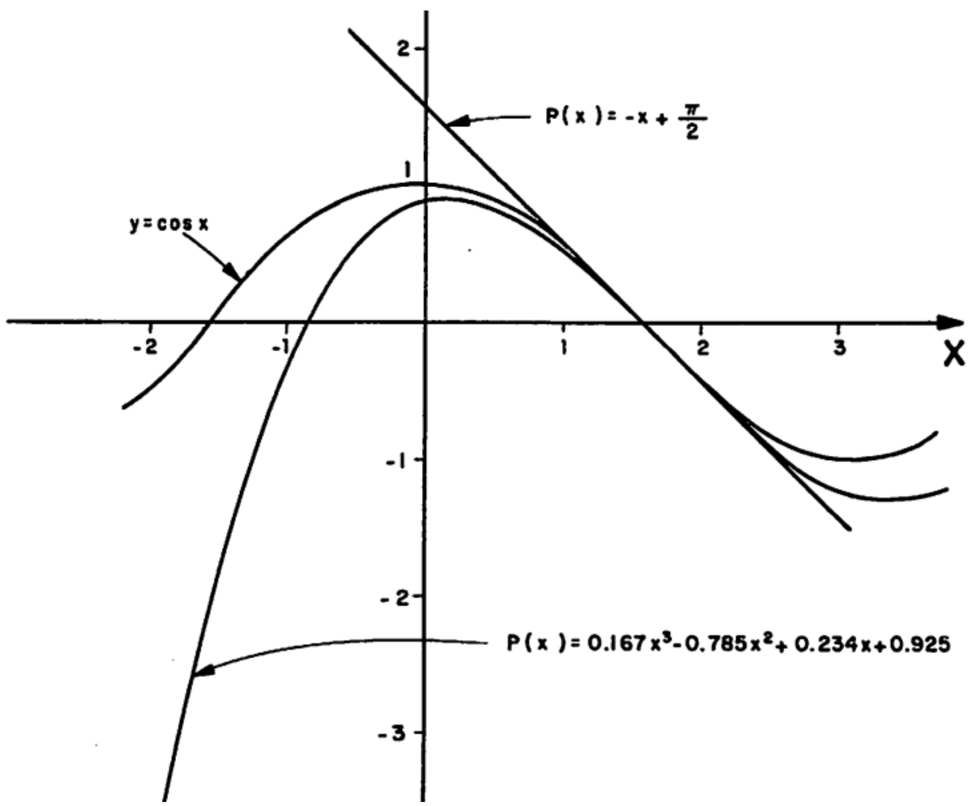

Figure 2: polinomio de Taylor grado 1 y 3; $f(x) = sen(x)$