Ecuaciones Diferenciales de Primer Orden

$y'(x) = x+2xy$ donde $y(0.5) = 1$, entre el intervalo $[0.5, 1.0]$ con paso de $h = 0.1$

Solucion ecuacion diferencial

$y'(x) = x + 2xy$ -> $\frac{dy}{dx} = x + 2xy$->$dy = x(1 + 2y)dx$->$\frac{dy}{1+2y} = xdx$->$\int \frac{dy}{1+2y} = \int xdx$

$\int \frac{dy}{1+2y} -> u = 1 + 2y$ ->$\frac{du}{dy} = 2-> \frac{du}{2} = dy$->$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}ln(u) = \frac{1}{2}ln(1 + 2y) + c1$

$\int \frac{dy}{1+2y} = \int xdx = \frac{1}{2}ln(1 + 2y) + c1 = \frac{x^2}{2} + c2 = ln(1 + 2y) = x^2 + c$->$e^{(x^2+c)} = 1 + 2y$->$\frac{e^{(x^2+c)}-1}{2} = y$

$\frac{e^{(x^2+c)}-1}{2} = y = \frac{e^{(0,25+c)}-1}{2} = 1-> e^{0.25+c} = 3-> ln3 = 0,25 + c-> c = ln3 - 0,25-> c = 0.847$

Solución por el metodo de serie de Taylor

$a = x_0 = 0.5$, $y(0.5) = 1$; $y'(x) = x + 2xy$

$$p(x) = \sum_{k=0}^{n} \frac{(x-a)^k}{k!} f^k(a)$$

$p(x) = \sum_{k=0}^{3} \frac{(x-0.5)^k}{k!} f^k(0.5) = \frac{(x-0.5)^0}{0!} f^0(0.5) + \frac{(x-0.5)^1}{0!} f^1(0.5) + \frac{(x-0.5)^2}{2!} f^2(0.5) + \frac{(x-0.5)^3}{3!} f^3(0.5) =$

$\frac{(x-0.5)^0}{0!} y(0.5) + \frac{(x-0.5)^1}{0!} y^{'}(0.5) + \frac{(x-0.5)^2}{2!} y^{''}(0.5) + \frac{(x-0.5)^3}{3!} y^{'''}(0.5)$

$y'(x) = x + 2xy-> y'(0.5) = 0.5 + 2(0.5)1 = 1.5$

$y''(x) = \frac{d(x+2xy)}{dx} = 1 + 2y + 2xy'-> y''(0.5) = 1 + 2(1) + 2(0.5)(1.5) = 1 + 2 + 1.5 = 4.5$

$y'''(x) = \frac{d(1+2y+2xy')}{dx} = 2y' + 2y' + 2xy'' = 4y' + 2xy'' = y'''(0.5) = 4(1.5) + 2(0.5)(4.5) = 6 + 4.5 = 10.5$

Reemplazo de evaluaciones de derivadas en el polinomio de Taylor

$p(x) = \frac{(x-0.5)^0}{0!} y(0.5) + \frac{(x-0.5)^1}{0!} y^{'}(0.5) + \frac{(x-0.5)^2}{2!} y^{''}(0.5) + \frac{(x-0.5)^3}{3!} y^{'''}(0.5)$

$p(x) = 1 + \frac{(x-0.5)^1}{1}(1.5) + \frac{(x-0.5)^2}{2}(4.5) + \frac{(x-0.5)^3}{6}(10.5) = \frac{7}{4}x^3 - \frac{3}{8}x^2 + \frac{9}{16}x + \frac{19}{32}$

Solución por metodo de Euler

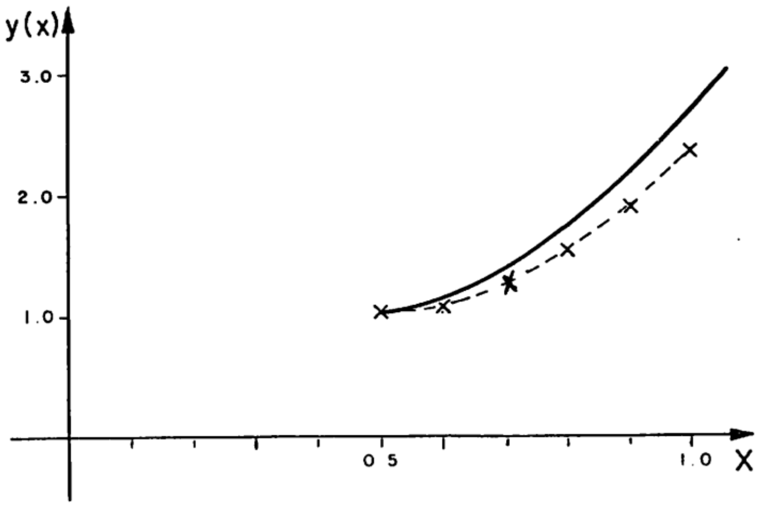
$a = x_0 = 0.5$, $y(0.5) = 1$; $y'(x) = x + 2xy$ ->$y'(x) = f(x, y)$ donde $f(x, y) = x + 2xy$ paso $h = 0.1$

$y_{i+1} = y_i + h.f(x_i, y_i)$

$y_1 = y_0 + hf(x_0, y_0) = 1 + (0.1)(0.5 + 2(0.5)(1)) = 1 + (0.1)(1.5) = 1.15$

$y_2 = y_1 + hf(x_1, y_1) = 1.15 + (0.1)(0.6 + 2(0.6)(1.15)) = 1.15 + 0.1(1.98) = 1.348$

$y_3 = y_2 + hf(x_2, y_2) = 1.348 + (0.1)(0.7 + 2(0.7)(1.348)) = 1.60672$
$y_4 = y_3 + hf(x_3, y_3) = 1.60672 + (0.1)(0.8 + 2(0.8)(1.60672)) = 1.94380$
$y_5 = y_4 + hf(x_4, y_4) = 1.94380 + (0.1)(0.9 + 2(0.9)(1.94380)) = 2.38368$



x	y(x)
0.5	1.00000
0.6	1.15000
0.7	1.34800
0.8	1.60672
0.9	1.94380
1.0	2.38368

Figure 1: Grafica y aproximación